μpscaling Small Models
Principled Warm Starts and Hyperparameter Transfer

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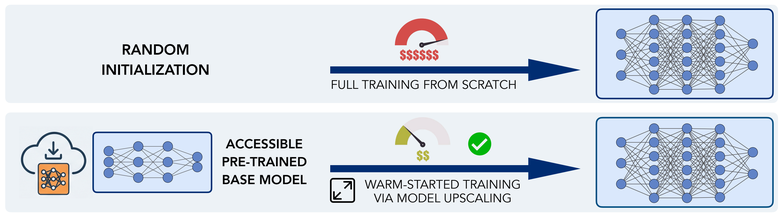
Model Upscaling



Prior work & limitations

- Based on function-preserving weight expansion [1].
- **The critical gap:** Performance is sensitive to hyperparameters, and tuning at the target scale is expensive.

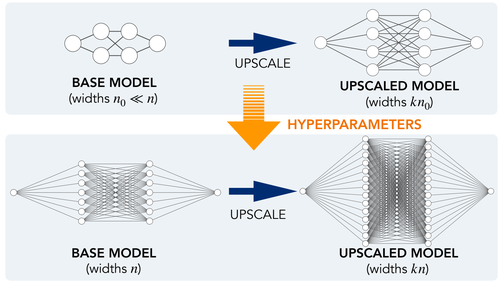
Our contribution:
a **theoretically-grounded** upscaling method (with respect to widths) that allows **hyperparameter transfer** via μP.

Proposed Upscaling Algorithm

Meta-algorithm 1 Upscaling procedure

Input: Base model checkpoint at width n with the optimizer checkpoint; expansion factor k.
Output: Trained upscaled model at width $N = nk$.

0. (HP tuning) Tune $\overline{\sigma_\Delta}$ and $\overline{\gamma^\uparrow}$ by running Steps 1-4 on a small proxy ($n_0 \to kn_0$, $n_0 \ll n$) and selecting values that minimize terminal training loss.
1. Construct an equivalent wider model of width N by duplicating and rescaling the weights.
2. Add noise to the weights following the μP initialization scaling, controlled by $\overline{\sigma}_\Delta$.
3. Transfer the base optimizer's internal states to the upscaled model's optimizer, analogously to the weights.
4. Train the upscaled model under μP with LR base constant $\overline{\gamma^\uparrow}$.



Maximal Update Parametrization (μP)

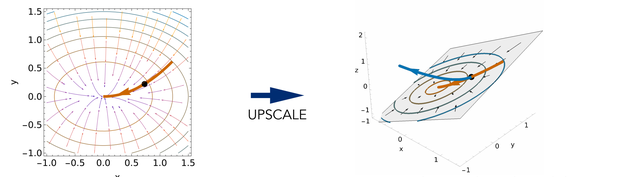
Weight type	Init Var. $\mathcal{N}(0, B\cdot\overline{\sigma}^2)$	LR $\gamma = C\cdot\overline{\gamma}$	WD (vanilla) $\lambda = D\cdot\overline{\lambda}$	WD (decoupled) $\lambda = \hat{D}\cdot\overline{\lambda}$	Extra HPs $\varepsilon = E\cdot\overline{\varepsilon}$	Output multiplier
Vector-like	1	n^m	n^{-1}	n^{-m}	n^{-1}	n^{-1}
Matrix-like	n_{in}^{-1}	$n_{\text{out}}^m n_{\text{in}}^{-1}$	$n_{\text{in}} n_{\text{out}}^{-1}$	$n_{\text{in}} n_{\text{out}}^{-m}$	n_{out}^{-1}	$--$

- m depends on the optimizer: $m = 1$ for SGD and $m = 0$ for Adam.
- ε stands for extra hyperparameters that require rescaling, such as the numerical stability constant `eps` in Adam.

- Yields optimal feature learning behavior in the infinite-width limit [2].
- Allows zero-shot hyperparameter transfer for training from scratch [3].

From Static Equivalence to Dynamic Equivalence

Theoretical guarantee 1:
Injecting zero noise ($\overline{\sigma_\Delta} = 0$) yields a widened model whose training trajectory **exactly** matches that of the base model.



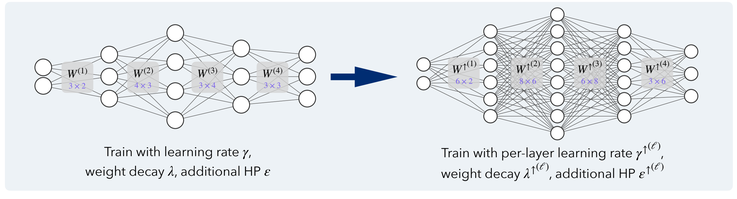
→ training trajectory of the base model

→ equivalent widened model
→ upscaled model with added noise

Theorem: For standard architectures (e.g., MLP, CNN, RNN, Transformer) and optimizers (e.g., SGD, Adam), the following models are ***dynamically equivalent*** (expresses the same function at each training step).

- The narrow **base model** trained with given HPs.
- The **widened model** (weights constructed in the following way) trained under μP with the same HP base constant as the base model.

Vector-like	$n \times d \to kn \times d$	`W.repeat_interleave(k, dim=0)`
Matrix-like	$n_{\text{out}} \times n_{\text{in}} \to k_{\text{out}} n_{\text{out}} \times k_{\text{in}} n_{\text{in}}$	`W.repeat_interleave(k_out, dim=0)` `.repeat_interleave(k_in, dim=1) / k_in`



Infinite-Width Limit and Hyperparameter Transfer

- μP-based HP transfer [3] applies to training from scratch only; the Tensor Programs framework [2,3] does not support the widening operation.
- We extend Tensor Programs to support mid-training upscaling.
- **Key idea:** Partition the width-kn activations into k groups of n neurons each. Each group uses only standard operations, placing upscaled training within the existing framework. In the infinite-width limit, activations now behave like i.i.d. k-dim Gaussian blocks with non-trivial covariance, rather than i.i.d. scalars.
- **Result:** Our algorithm's design choices are precisely those that keep the entire training process (including mid-training upscaling) under μP.

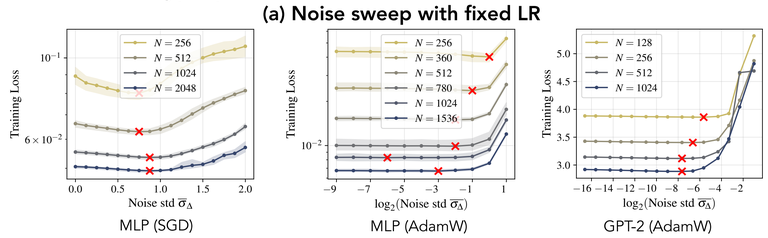
Theoretical guarantee 2:
In the infinite-width limit, the injected noise is both **safe** (base-model signal preserved) and **useful** (noise's contribution neither vanishes nor explodes).

Theoretical guarantee 3:
Hyperparameters $\overline{\sigma}_\Delta$ (noise level) and $\overline{\gamma^\uparrow}$ (upscaled model LR) can be tuned efficiently via μP-grounded **hyperparameter transfer**.

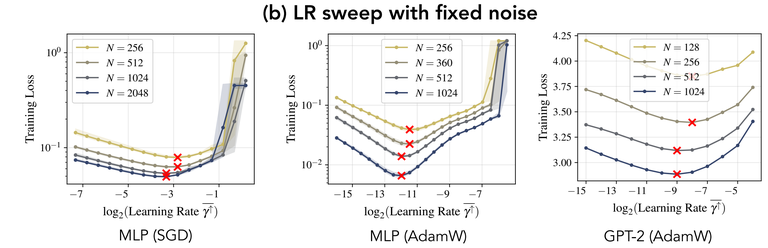
Experiments

Validation of hyperparameter transfer

(a) Noise sweep with fixed LR

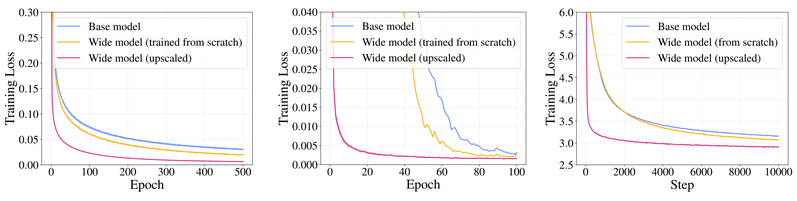


(b) LR sweep with fixed noise



Upscaling accelerates training

MLP (AdamW) on Forest Cover Type	ResNet (SGD) on CIFAR-100	GPT-2 (AdamW) on FineWeb
width $500 \to 2000$	$2\times \to 4\times$ model	width $1920 \to 3840$
tuning on width $100 \to 400$	tuning on $0.5\times \to 1\times$ model	tuning on width $192 \to 384$
$23.6\times$ tuning-FLOP reduction	$16.0\times$ tuning-FLOP reduction	$48.2\times$ tuning-FLOP reduction



Conclusions and Future Work

Key contributions

- Provides the first rigorous HP transfer framework for model upscaling, applicable to general architectures and optimizers.
- Extends the theoretical foundation from static to dynamic equivalence.
- Connects model upscaling to the μP infinite-width theory.

Future work

- Extend to depth-wise and joint width-and-depth upscaling.
- Understand generalization behavior under upscaling (current framework addresses training dynamics only).
- Validate at industry-scale settings, where larger target-to-tuning ratio n/n_0 yields greater HP tuning cost savings.

References

[1] Chen, Tianqi, Ian Goodfellow, and Jonathon Shlens. **"Net2Net: Accelerating Learning via Knowledge Transfer."** *ICLR (2016).*
[2] Yang, Greg, and Edward J. Hu. **"Tensor programs IV: Feature learning in infinite-width neural networks."** *ICML* (2021).
[3] Yang, Greg, et al. **"Tensor programs V: Tuning large neural networks via zero-shot hyperparameter transfer."** *NeurIPS (2021).*